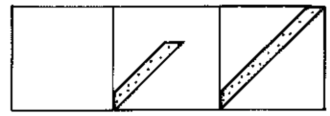
Multiplication Designs

Why

To practice recognition of multiplication facts and to see connections between numbers and their multiples

► *Recognizing multiples quickly is important in addition and subtraction, reducing fractions, and long division, as well as in algebra.* ◄



Grade Level

TOOLS

Several multiplication tables

Crayons or marking pens

How

- ☐ Pick a number from 2 to 12 and write it on top of your multiplication chart.
- ☐ Above your multiplication table, write all of the multiples of the number you chose. Continue your list until you get to 144, or as close as you can.
- ☐ For each multiple, find all the places it occurs on the table; color in those squares on your chart (see example below).
- ☐ Compare your design with one someone did for another number —or choose a different number from 2 to 12 and make a new design.
- ☐ Discuss the patterns made by different numbers. Why do 8 and 9 have more complex patterns than 7?

MULTIPLICATION DESIGNS

(8)

8 16 24 32 40 48 56 64 72 80
88 96 104 112 120 128 136 144

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

MULTIPLICATION DESIGNS

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
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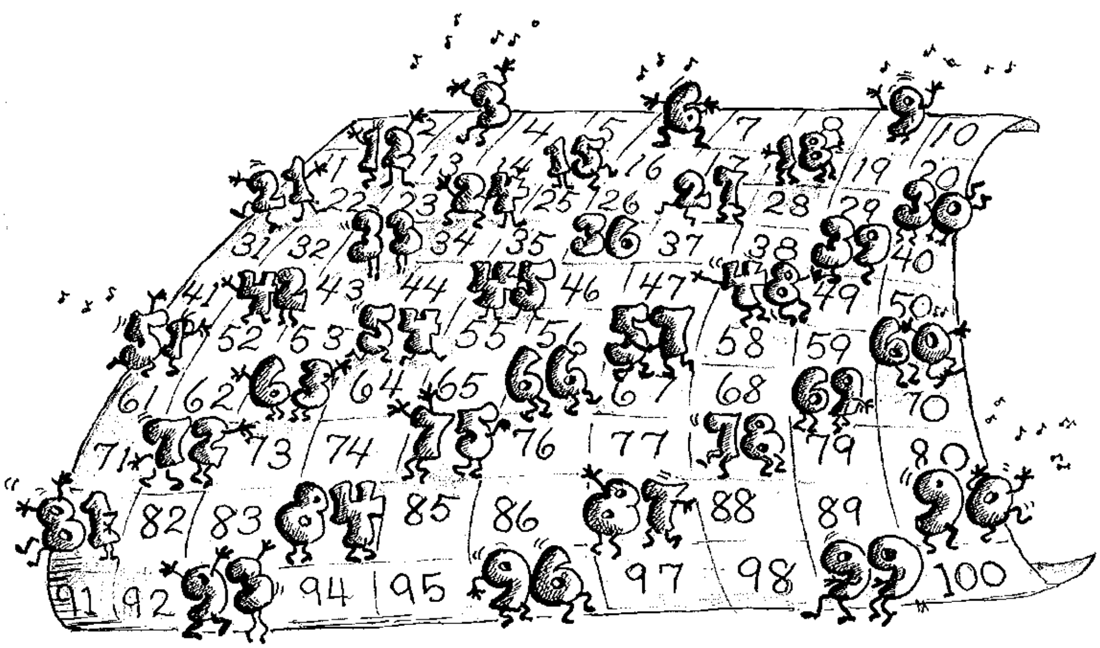
Algebraic reasoning

Hundreds Charts to Descartes I

"Skip counting" (counting by 2's, 3's and so on) on a hundreds chart results in interesting patterns on the charts themselves. These patterns can be converted to graphs on the *Cartesian plane*. (If you are unfamiliar with plotting points on a graph, you may want to try *Getting to the Point* before you do this activity.)

René Descartes was the first person to write about the idea of using a coordinate system to represent algebraic equations. A graph or "picture" of an algebraic equation can greatly simplify solving some problems. For example, instrument calibration and conversions, such as engine rpm to miles an hour, can be done using graphs.



ACTIVIDADES DE LOS MATEOYCIENTINA

MATERIALS

- hundreds charts (see page 252)
- counters and colored pens
- graph paper (page 247 or 248)
- calculators

WHAT'S THE MATH?

Coordinate graphing; multiples; converting patterns to algebraic equations and graphs.

How

Creating and Recording Patterns

• Work with a partner or two other people.

• Use the addition constant feature on a calculator to skip count by 2's. (To do this on most basic calculators, you enter 2 + = = = … . Check the calculator direction pamphlet if this does not work. Ask your partner to place a marker on the hundreds chart for each number as you call them out. (If you don't have calculators, you can count by 2's directly on the hundreds chart.)

• Record your results in a table like the one to the right.

n: number of marker	V: value of number landed on
1st	2
2nd	4
3rd	6
4th	8
.	.
.	.
.	.

• Does anyone in your group have an idea for an equation or rule that tells how to find the value of the number you land on if you know the number of the marker? If the marker number is 4, what is the value? If it is 6, what is the value? What do you do to the marker number to get the value?

• Can you predict what the value will be for the 14th marker? For the 26th marker? How did you figure these out?

• What if you wanted a more general rule or equation for finding the value for any marker, for example the "nth" marker? How could you find the value of the number you would land on? How could you write what you did? You might say: the value is twice the marker number. But what would a mathematician do? They like to write equations, which are really abbreviations of the sentences we write in words.

If you think about any marker number, n, what do you do to n to get the value, v? Some people might give the equation $v = 2n$ right away, while others might need to test out more specific examples. It doesn't mater what letters you use for the unknown, as long as you are consistent.

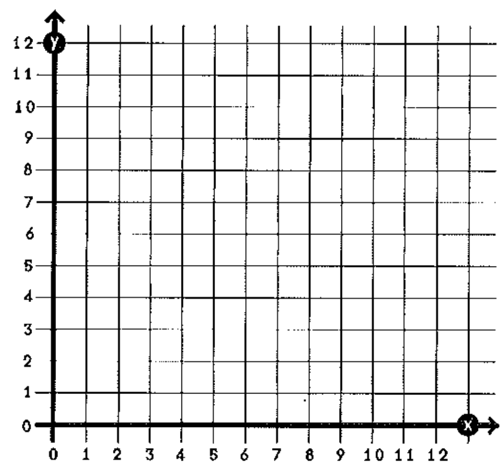
Hundreds Charts to Descartes I

Making a Graph

• Now let's look at a way to make a pictorial representation of our table of the pattern of skip counting by 2's.

• Use graph paper like the one below with lines that are numbered along the horizontal and vertical axes or make your own.

Note: When negatives appear in front of the numbers, you move left on the horizontal axis and down on the vertical axis.



• Think of each entry in your table as an ordered pair: (1, 2), (2, 4), (3, 6), and so on. The first number tells us how far to move to the right from 0 along the horizontal axis. The second number tells us how far to move up from 0 on the vertical axis.

• Before you plot a point for each ordered pair in the table, make a prediction about what pattern the points will make. Then plot the points.

• What pattern are you getting? Do you think it's OK to connect the points to make a straight line? If you do that, it looks like the line would go through points like (0.5, 1) and (1.5, 3). Do they fit the pattern in the table?

If you extend the line, can you find other points that would fit from the line? Now you have a graph of $v = 2n$ (or $y = 2x$).

• If you were to count by 3's, 4's, or 5's, what do you think will happen? Will the graph be a straight line again or a curve? Will the graph be above or below the line that was generated when we counted by 2's? Explore these questions with your family. Plot the results for each graph in a different color. Be ready to share your results.

Even though using n and v in our equations might help us remember what we were counting or looking at originally, mathematicians would commonly label these graphs as: $y = 3x$, $y = 4x$, and $y = 5x$, and so on, where x represents the number measured on the horizontal axis and y represents the number measured on the vertical axis.

• Here are some questions to think about: What will the graph for the equation $y = x$ look like? What rule or function will give us a graph or straight line below $y = x$ (closer to the horizontal axis)? ■

Doubling Bean Boxes

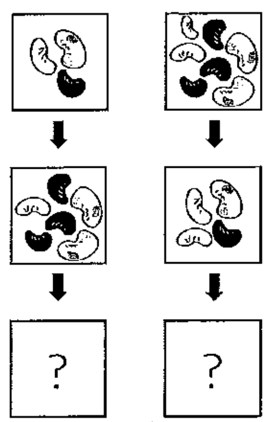
3–6

MATERIALS
scratch paper for drawing boxes
45 beans

You may want to do Bean Boxes before trying this activity.

How

- Work with your child, taking turns and keeping a record of each step.
- Begin with some number of beans, divided *unequally* into two boxes. For example, start with 10 beans, divided into one box of 3 beans and one box of 7 beans.
- Double the size of the smaller group by taking 3 beans from the larger group, leaving new numbers of 4 and 6 beans in each box.



- Using the new numbers of beans in the boxes, repeat the process. In other words, double the size of the new smaller group by taking beans from the larger group. What are your new numbers?
- Continue the process. What is happening?
- But that's not all! Try this again but this time begin with 5 and 6 beans in each box.
- Keep track of each step. Compare what happens when you start with each new set of numbers.

This is about
developing an understanding of factors and multiples by using logic.

- Try other pairs of numbers such as 3 and 5, 7 and 1, and 2 and 14.
- Try larger pairs of numbers such as 2 and 24, 16 and 4, and so on.

More Questions

- How many moves are made in each case before the smaller group becomes the larger?
- Look for patterns with all the possible combinations for the numbers 7 and 20. What patterns do you find?
- Do number sets that are close together and those far apart give different results?
- Sometimes you reach a point where no moves can be made. How many moves did it take to get there?
- Which numbers are unusual?
- Do the factors of a number matter? How do you know?

Button Boxes

This is About

- being friendly with numbers
- practicing adding

You Will Need

- buttons
  or small blocks
  or bottle-caps
  or design your own cardboard buttons
- if you like, you might cut apart an egg carton to make the boxes shown

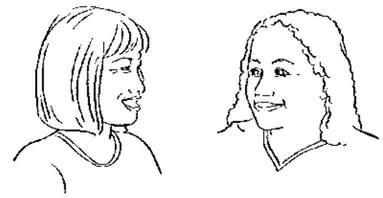
Getting Ready

Part of the object of this activity is to practice combining small numbers to make larger numbers in an informal way, with things we can move around.

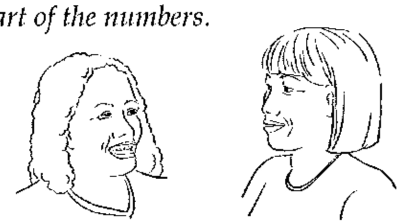
When you have played the game a few times, make up different sets of "boxes," to practice other number combinations.

With the boxes in this activity, at least two of the suggested combinations will not be possible. What are they?

Why buttons? Why can't we just write down the numbers and have the kids add them, the way we did when we were in school?



We move buttons around and match each button to its own box, so they are part of the numbers.



When do they learn about adding numbers together?

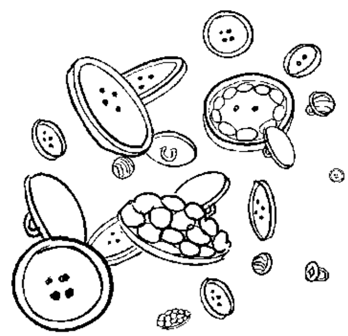
When they find that they can combine a 2-box and a 3-box to make 5, that's adding!

ACTIVITY

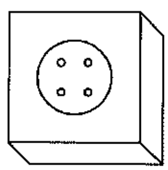
The Story

You're helping with the button booth at the school fair. The buttons are very special. Each button must have its own space in a box. You can't sell a box with any empty spaces!

Since you can't sell a box with any empty spaces, if you put a button into one space of a box, you also must put buttons into the other spaces of that box.

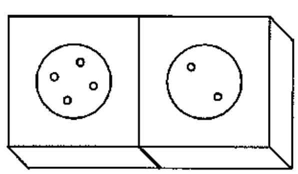


Here is a practice box. How many buttons could you put into this box?



(Only one, of course!)

Here's another practice box. How many buttons would it hold?



(two buttons)

If you used both of these boxes, how many buttons could you put in?

On the following pages is a set of boxes and a list of orders to fill. How many of the orders can you fill with those boxes?

Button Boxes (continued)

These are the button boxes you have for the button booth orders at the school fair. You may use as many boxes as you need for each order.

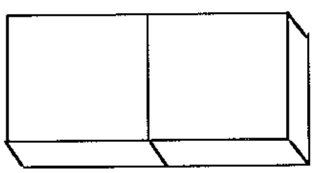
Talk together about each order.

Each button must have its own space.

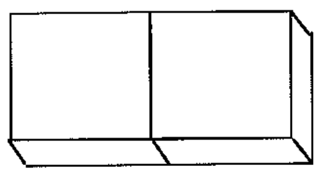
You can't sell a box with any empty spaces.

On the next page are some button orders.

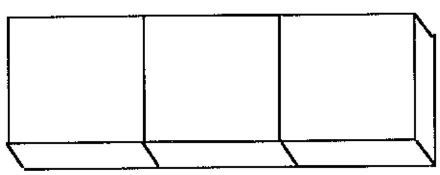
Box A



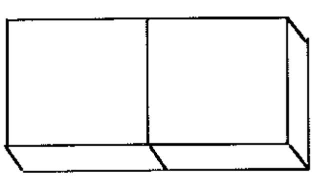
Box B



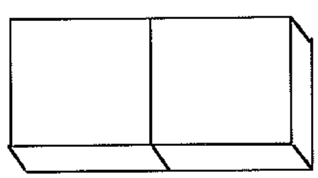
Box C



Box D



Box E



How many buttons will go in:

Box A? ______________________________

Box B? ______________________________

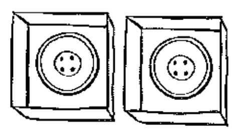
Box C? ______________________________

Boxes A and B? ________________________

Boxes C and D? _______________________

Boxes C, D and E? _____________________

Make up some of your own combinations.



Here are some orders. Which boxes would you use for these orders?

1 button (can't be done!) __________

2 buttons ________________

3 buttons ________________

4 buttons ________________

5 buttons ________________

6 buttons ________________

7 buttons ________________

8 buttons ________________

9 buttons ________________

10 buttons _______________

11 buttons _______________

12 buttons _______________

Is there more than one way to fill some of these orders?